Discrete spectrum of nonlinear modes in weakly nonlocal problems: a mechanism to emerge

G.L.Alfimov[1], E.V.Medvedeva[1], D.E.Pelinovsky[2]

[1] National Research University of Electronic Technology MIET

[2] Department of Mathematics, University of McMaster, Hamilton, Canada

email: galfimov@yahoo.com

Abstract:

We discuss a hypothesis on existence of a countable set of heteroclinic orbits connecting saddle-center points (also called embedded solitons" in some applications). In short, it can be described as follows. Let a system of differential equations depend on some external parameter " that defines a singular perturbation. Assume that the system has a heteroclinic orbit for $\epsilon = 0$ and that the corresponding solution can be analytically extended into upper complex half-plane with the closest to the real axis singularities given by a pair of points $z = \pm a + ib$. Then there is a countable set of heteroclinic orbits for the singularly perturbed system corresponding to the discrete set of values ϵ, $\epsilon = \epsilon_1, \epsilon_2, \epsilon_3, \ldots$ such that $\epsilon_n \to 0$ as $n \to \infty$ and

$$\epsilon_n \sim \frac{b}{\pi/2 + n\pi + \phi_0},$$

where ϕ_0 is a constant. We illustrate this statement by numerical results for several nonlinear and weakly nonlocal problems of various physical origin.